Whither Time's Arrow?

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In our everyday lives we have the sense that time flows inexorably from the past into the future; that time has a definite direction; and that the arrow of time points towards a future of greater entropy and disorder. But in the microscopic world of atoms and molecules the direction of time is indeterminate and ambiguous.

The true nature of time is somewhat mysterious. We creatures of time sense that time flows, that the present is real, the past remembered, the future anticipated. Rain falls; rivers flow downhill; mountains erode; we are born, grow old, and die; order decays into chaos. In our everyday experience, time has a definite direction, and the arrow of time points towards the future. But despite the blatant time-asymmetry of our everyday lives, the accepted, fundamental theories of physics are time symmetric[1]. Neither Newtonian mechanics, special or general relativity, quantum mechanics, nor quantum field theory picks a preferred direction in time, anymore than these theories picks out a preferred direction in space. Physics provides no objective reason to believe that our present is in any way special, or more real than any other instant of time. Why then does the past appear different from the future?

The one fundamental theory that does pick out a preferred direction to time is the second law of thermodynamics. The second law asserts that the entropy of any closed system, the Universe included, tends to increases towards a state of maximum entropy called thermal equilibrium. This provides an orientation to time, colloquially time's arrow, and this arrow points towards the future, which is the direction of increasing disorder. Other time asymmetries, such as our inability to remember the future, appear to be due to this fundamental entropic asymmetry.

Embracing the second law does not resolve the central enigma(s) of time; it simple opens new vistas of ignorance. Entropy increases today because it was lower yesterday. If we look back far enough, we discover that about 14 billion years in the direction we naively call the past, the Universe was small, dense and homogenous, and the total entropy was very, very small. The entropy of the Universe has been increasing ever since, and because the Universe is currently nowhere near a state of thermal equilibrium, entropy will continue to increase into the indefinite future. Thus, part of the mystery of time is cosmological and related to the birth of the Universe (where, incidentally, the known laws of physics are not to be trusted). But new mysteries become apparent. For instance, why are there 3 spacial dimensions, but only one temporal dimension? And why is the Universe asymmetric in time, but on large scales spatially homogeneous and symmetric? Why does entropy increase into the future, but not towards (for instance) the right?

Another part of the mystery of time has become apparent and been be explored with physics developed over the last 15 years or so. Measuring an interval of time simple requires a clock, in principle an entirely mechanical, deterministic mechanism. But assigning a direction to time requires that we observe a change in entropy. But it is important to real-

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[1] More precisely, the laws of reality appear to be symmetric under the combined symmetry of time inversion, parity inversion and charge inversion. However, the violations of charge-parity symmetry are negligible except in some areas of high-energy particle physics, and are neglected in this discussion.

ize that the common statement of the second law of thermodynamics, that entropy increases, is incomplete. A more precise statement is that the entropy S of any isolated system (including the Universe) increases in the direction of time we call the future, *on average*.

$$\left\langle \Delta S \right\rangle \geq 0$$

The caveat, on average (indicated by angled brackets in the equation), is often omitted or implied. This is partly because this extra condition is irrelevant on a macroscopic scale. If the change in entropy is large, then it is virtually guaranteed to be positive. But in general, if I perform an experiment and the change in entropy is small, then entropy could increase or it could decrease. If I repeat the experiment many times, any occasional decrease in entropy must be balanced by an increase in entropy observed during a different repetition, since the second law requires that change in entropy, averaged across many repetitions, is always zero or positive.

Since entropy can both increase and decrease, this leads to an interesting conundrum. How does one determine the the orientation of time's arrow for a given process? Since for any single realization of a process entropy can either increase or decrease, the orientation of time isn't absolute, but for small systems becomes nebulous and difficult to resolve. For instance, suppose that I am given a movie of a ball tossed into the air. Can I determine which way to feed the movie into the projector without looking at the time marks on the film? Can I reliable assign an arrow of time to the action depicted? Suppose only the free flight of the ball is shown, as it rises, slows under the force of gravity and descends again. Due to air friction, the ball will lose energy, and descend slightly slower than it rose. Some of the kinetic energy of the ball is converted into random motion of the air molecules, thereby increasing the entropy of the Universe. Provided this effect isn't too small, I can, in principle correctly determine the original orientation of the film.

However, we will encounter difficulties in orientating the film if the energy dissipated by the ball is very small (We will address how small shortly). Suppose, for instance, that we examine only a few frames separated by very short time intervals. Over a short enough time the energy of the ball need not decrease. The ball is continuously buffeted by the random motion of the surrounding air molecules, and by chance it could so happen that a large number of molecules hit the ball at high speed from behind, minutely increase the ball's energy. Viewed with a fine enough time resolution, the energy of the ball does not decrease monotonically with time. Rather, the energy fluctuates moment to moment, mostly decreasing, but occasionally increasing. Since assigning a direction to time requires a dissipation of energy and a corresponding increase in entropy, this means that the orientation of time's arrow is ambiguous[2] on short time scales.

In the essentials, the preceding thought experiment has actually been performed using modern experimental finesse. The ball in question is a latex bead about a micron wide, suspended in water, rather than air. Instead of gravity, the bead is pulled through the aqueous environment using an optical trap, the light pressure of an intense laser beam. And the previously described scenario can be observed; energy can be transfered from the optical trap to the bead, and thence to the environment, increasing the entropy of the water. Or, by random chance, energy can be sucked away from the bead, reducing the entropy of the aqueous environment. The total entropy increases on the average, but can fluctuate both up and down, moment to moment.

A variety of other experiments on more complicated systems have also been performed in the last few years, ushering in a new era of quantitative, experimental thermodynamics of small systems. An important example is illustrated below. A single RNA molecule is attached between two micron sized latex beads with DNA linkers. One bead is captured in an optical laser trap that can measure the applied force on the bead, and the other bead is attached to a piezoelectric actuator. Initially, the RNA molecule is in thermal equilibrium in a folded, compact configuration. The inter-bead separation is then increased, pulling the molecule apart into an extended, unfolded state. Due to thermal fluctuations, the amount of energy required to pull apart the molecule varies from one repetition of the experiment to the next.

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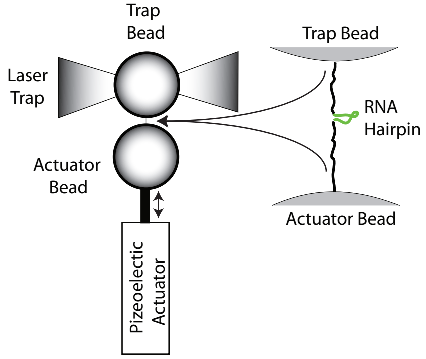
[2] *Time flies like an arrow. Fruit flies like a banana.* – Groucho Marx

Consequently, the change in entropy is different from one realization of the experiment to the next, and sometimes the change in entropy is negative.



Relatively recently, it was realized that the random variations in observed entropy change are governed by a rather simple symmetry. Any experiential protocol has a corresponding reversed protocol. If I drag a bead to the left, then in the reverse protocol I drag the bead to the right. If the forward protocol is the forced unfolding of an RNA hairpin, then the conjugate reverse protocol starts with the RNA hairpin at thermal equilibrium in an extended, unfolded state, and then reduces the inter-bead separation, allowing the molecule to refold. The probability of observing a particular total change in entropy ΔS during the forward protocol F is related to the probability of observing a negative dissipation of equal magnitude during the conjugate reverse protocol,

$$\frac{P_F(+\Delta S)}{P_R(-\Delta S)} = e^{\Delta S} \ .$$

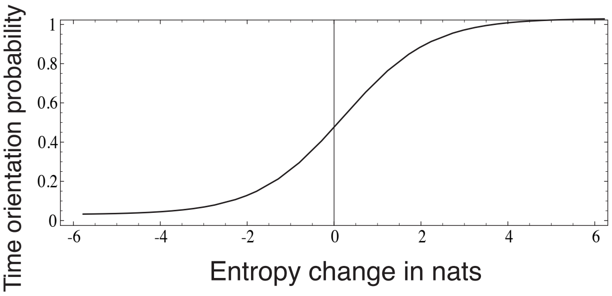
This, and similar, closely related relations, are known as *entropy fluctuation theorems*. An important corollary is that it is possible to rewrite the second law as an exact equality, the Jarzynski identity, rather than the traditional inequality.

$$\ln \left\langle e^{-\Delta S} \right\rangle = 0$$

These relatively recent, and initially somewhat surprising, generalizations and restatements of the second law are connected to a whole host of novel insights into the behavior of small, molecule scale systems away from thermal equilibrium. They also quantify the relation between time’s arrow and the dissipation. Dissipation breaks time symmetry, and conversely, any breaking of time symmetry implies a change in entropy. Suppose that we are given a few seconds of movie showing the aforementioned ball pulled through water. Further, suppose the movie has perfect resolution, capturing the movement of every molecule in the fluid and every atom in the ball. And, as before, we wish to determine the direction of time, the original orientation of the movie. All of the molecular details turn out to be irrelevant. The only thing that matters is the increase in entropy, which we can calculate given the energy required to drag the ball through the fluid and the temperature of the environment. We watch the movie and observe a change in entropy of ΔS, which could be positive or negative. The probability that the movie is correctly oriented is

$$p = \frac{1}{1 + e^{-\Delta S}} \ .$$

This follows from the entropy fluctuation relation and normalization. This equation is illustrated here:



If the change in entropy is large and positive, then we certainly have the dynamics correctly orientated. If large and negative, the original time direction was opposite that of the current orientation. But intermediate entropy changes could be generated from either time orientation, and we cannot say with certainty which way time flows. Given a small change in entropy, the best we can say is that the future is probable in the direction of increasing disorder.

We need to take a small digression to discuss units of entropy. The thermodynamic entropy of a glass of water, of other such large everyday object, is typically measured in Joules per Kelvin per mole, but this is largely an historical accident. Most common, everyday units are arbitrary, human created conveniences. But entropy is unusual among fundamental observables in that its natural units are actually a useful size. Entropy is (crudely speaking) the logarithm of the total number of alternative possibilities, $S = \log N$. The unit of entropy is determined by the base of the logarithm, of which there are only two natural choices. Either we use base 2 logarithms and measure entropy in bits ('BInary digiTS), or natural logarithms ($\ln$, base e), and measure entropy in nats ('NAtural digiTS'). 1 nat ≈ 1.44 bits $\approx$ 8.14 J K^{-1} mol^{-1}. For comparison, the disorder of water at room temperature and pressure is about 7 nats, or 10 bits per molecule, and the average thermal kinetic energy of a single molecule is equivalent to 1.5 nats, or about 2 bits. Bits have the advantage of being more intuitive, since a single bit is the maximum information of a single yes or no, true or false question. But, mathematically, it is most natural to express the equations of statistical thermodynamics in nats, as is done here.

Returning to the previous equation, we can see that for time's arrow to be ambiguous the absolute change in entropy must be truly microscopic, on the order of only few bits. For instance, if the change in total entropy is 4 nats (about 6 bits) then there is a better than 98% chance that we have the correct time orientation. Conversely, large drops in entropy are rare. A modest decrease of 14 nats (entropy equivalent of 14 $k_{\mathrm{B}}T$'s of thermal energy, or about 20 bits) will occur less than one time in a million. A macroscopic decrease in entropy is unthinkable.

One important lesson here is that the orientation of time's arrow is in no way absolute; it is contextual and depends on the local environment. For instance, it has been suggested that despite appearances, maybe our Universe really is time symmetric, with a low entropy big crunch in our distant future. If we are located close to one low entropy boundary of the universe then we would not yet notice any effect from the other, distant low entropy boundary. In such a universe time's arrow would point firmly towards the temporal mid point when close to either end, but would become indeterminate, flipping back and forth in the central region of time.

To summarize, we can learn a great deal about the nature of time by contemplating the behavior of microscopic molecular systems driven away from thermal equilibrium. Time and entropy are intimately linked. The directionality of time, and the distinction between past and future, are not absolute. To assign a direction to time requires an increase in entropy. But if the entropy increase is very small the direction of time is uncertain, and without an entropy gradient time would have no orientation at all. Modern statistical physics has placed these observations on a solid, quantitative footing.

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Further reading: A more detailed and technical discussion about the difficulties of orientating time's arrow can be found in *Length of time's arrow* [1] and *The footprints of irreversibility* [2], and citations therein. The term "time's arrow" was coined by Arthur Eddington, and his book remains a good introduction to the subject [3]. Huw Price provides a good discussion of the thermodynamic arrow of time and how is underlies other manifestations of time asymmetry [4]. For an introduction to experimental microscopic thermodynamics see [5]. For a recent overview of fluctuation theorems and related advances in non-equilibrium thermodynamics see [6].

[1] E. H. Feng and G. E. Crooks. Length of time's arrow. *Phys. Rev. Lett.*, 101(9):090602, Aug 2008.

[2] A. Gomez-Marin, J. M. R. Parrondo, and Christian Van den Broeck. The footprints of irreversibility. *Europhys. Lett.*, page 50002, Jun 2008.

[3] A. S. Eddington. *The nature of the physical world*. Cambridge University Press, Cambridge, 1928.

[4] H. Price. *Time's arrow and Archimedes' point*. Oxford University Press, New York, 1996.

[5] C. Bustamante, J. Liphardt, and F. Ritort. The nonequilibrium thermodynamics of small systems. *Phys. Today*, 58(7):43– 48, Jul 2005.

[6] C. Jarzynski. Nonequilibrium fluctuations of a single biomolecule. In *Controlled Nanoscale Motion*, volume 711 of *Lect. Notes Phys.*, page 201. Springer, Berlin, 2007.