

Comparison of the roughness scaling of the surface topography of Earth and Venus.

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Abstract

We report the scaling behavior of the Earth and Venus over a wider range of length scales than reported by previous researchers. All landscapes (not only mountains) together follow a consistent scaling behavior, demonstrating a crossover between highly correlated (smooth) behavior at short length scales (with a scaling exponent $\alpha=1$) and self-affine behavior at long length scales ($\alpha=0.4$). The self-affine behavior at long scales is achieved on Earth above 10 km and on Venus above 50 km.

It has been suggested based on a number of studies that horizontal transects of mountain ranges follow self-affine scaling[1–16]. However, quite different values of the scaling exponent have been proposed. Typically, it has been found that scaling exponents at shorter length scales are higher than scaling exponents at longer length scales. A consistent picture of the scaling behavior of the Earth’s topography accounting for these diverse results has not yet been provided. We find, by studying a larger range of length scales, that all landscapes (not only mountains) together follow a consistent scaling behavior, demonstrating a smooth crossover between a relatively smooth, correlated landscape at short length scales (with a scaling exponent $\alpha = 1$) and self-affine behavior at long length scales ($\alpha = 0.4$). Contrary to expectations that mountain ranges are self-affine over all scales, we find that only above distances of 10 km does a self-affine scaling with a unique exponent $\alpha = 0.4$ apply. The topography of Venus shows a similar crossover at a larger length scale of 50 km to self-affine scaling that extends to 7×10^3 km.

To investigate the scaling of topography, it is necessary to analyze the Earth’s surface over a wide range of lengths. Topographic data for the US have been collected by the US Geological Survey[17]. While data are available for other regions of the Earth, this is the highest resolution data over the widest area publicly available. Elevations for the contiguous 48 States are provided in 1 degree square blocks (approximately 100 km by 100 km), at a horizontal resolution of 3 arc-seconds (≈ 100 m). This provides about 10^9 data points covering an area of $7.5 \times 10^6 \text{km}^2$.

For a self-affine landscape the standard deviation of surface elevation W should scale with sample size (linear dimension) L as

$$W \sim L^\alpha, \tag{1}$$

where α is the roughness scaling exponent. A number of methods have been used to obtain the scaling behavior. We calculate W for square areas of increasing edge length L , and then average over the entire region. Averages of the scaling behavior of linear transects taken through the same dataset provided essentially similar scaling behavior. We use real space rather than Fourier space evaluation of the standard deviation since Fourier transforms impose periodic boundary conditions and thus discontinuities which result in distortions of the scaling behavior at short length scales[18].

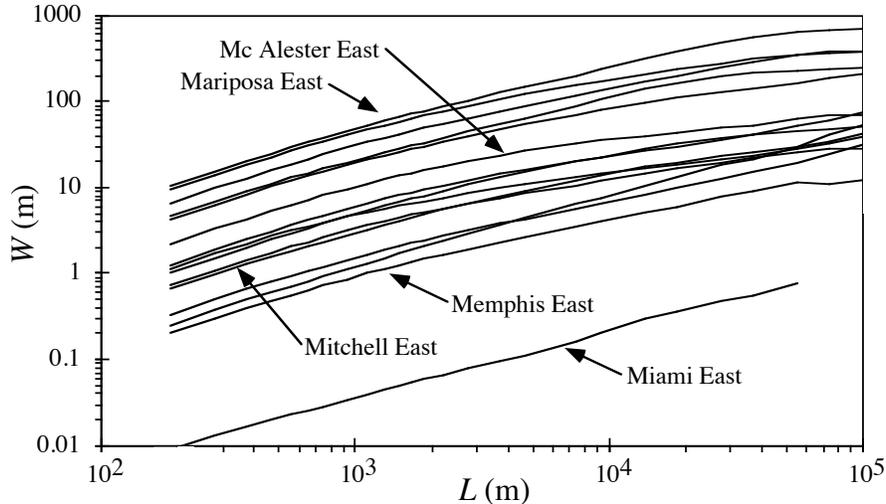


FIG. 1: Standard deviation W of elevations in randomly selected 100 km by 100 km regions of the US. Several locations with very different topographies, have been labeled. Mariposa East covers part of the Sierra Nevada mountains, and contains Kings Canyon national park. McAlester East is in eastern Oklahoma. Mitchell East is in southeast South Dakota. Memphis East covers the southwest corner of Tennessee, and is bisected by the Mississippi river. Miami East covers part of the Florida everglades. We calculate W for square areas of increasing edge length L , and then average over the entire region. The curves are nearly parallel, illustrating the remarkable universality of roughness scaling despite the wide variation in absolute roughness. The larger variation at larger values of L can be attributed to averaging over fewer samples at these lengths.

Figure 1 shows the calculated values of $W(L)$ for a random selection of 1 degree by 1 degree regions. The curves represent a great range of landscapes. The top curve, Mariposa East (37-38 N, 118-119 W), covers part of the Sierra Nevada range including Kings Canyon National Park in California. This extremely mountainous terrain has a high point at 4341 m above sea level. The bottom curve in Fig. 1 represents data from the Everglades, an extremely flat terrain (Miami East, 25-26 N, 80-81 W). Intermediate curves are indicated in the figure. This selection of landscapes covers 3 orders of magnitude in roughness.

Remarkably, it can be seen that most of the landscapes have similar scaling behavior. In particular, scaling properties are not limited to mountainous regions, but apply to plains and flatlands as well. Scaling exponents calculated for each region from $L=40$ km to 120 km are broadly distributed with a mean of 0.42, and a standard deviation of 0.2. There appears to be no significant correlation ($r = 0.17$) between α and the roughness of the

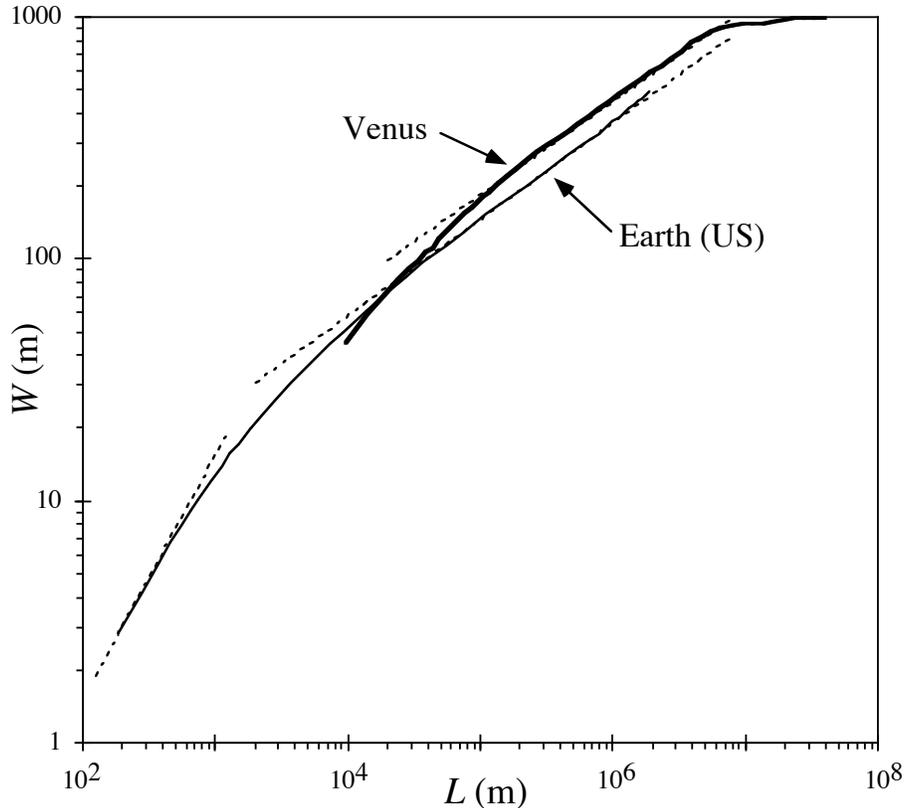


FIG. 2: Standard deviation of elevations of the entire continental US (thin solid line) and of the surface of Venus (thick solid line). The US results are calculated using square areas as in Fig. 1. Due to surface curvature of Venus at longer length scales, we calculate the Venus results for circular areas of diameter L . Square areas produce similar results. For the US data, we find a crossover between highly correlated smooth behavior ($\alpha=1$) at the shortest length scales, and self-affine behavior at the longest scales. The upper dashed line is fit to the data between 100 and 1000 km and has an exponent $\alpha=0.40$. The lower dashed line has an exponent of $\alpha=1.0$. The Venus data have a similar crossover. The corresponding dashed line is fitted between 200 to 2000 km and has an exponent of $\alpha=0.39$.

landscape (determined by the value of W at $L = 120$ km). There is a weak correlation between geographically neighboring regions.

Figure 2 shows results for the entire continental US. There is a variation of α with length scale consistent with previous studies. The asymptotic scaling regime is reached only at approximately 5 km and the scaling exponent (Fig. 3) has a value of 0.4. This exponent corresponds to the value predicted for scaling of self-affine surfaces that are growth fronts[19].

It is not obvious, however, that these models should apply to the Earth's topography. Nevertheless, scaling arguments suggest that essentially all dynamic surfaces with up-down asymmetry are part of a universality class with this scaling exponent. Correlations exist up to the maximum length scale of the data. However, at shorter length scales the landscape is smooth and approaches a scaling exponent of $\alpha=1$. This scaling exponent is consistent with a highly correlated affine surface or even a linear surface on short length scales. It reflects the smoothness of both mountain slopes and other terrain at these length scales.

Our results are consistent with the overall pattern of observations by other researchers. For example, Dietler and Zhang[10] performed an analysis for Switzerland, an area of $7 \times 10^4 \text{km}^2$ with a lower resolution of 250 m, and by fitting a single line to values below 5 km they obtained $\alpha \approx 0.57$. They also indicate that higher resolution data led to still larger slopes. Above this scale their data suggested $\alpha = 0.27$, whose low value may be attributed to the small number of independent samples, consistent with the range of values found in the data we studied. Turcotte's analysis[16, 20] of the large scale Earth scaling behavior yielded a value of $\alpha=0.5$. His analysis combines bathymetric and topographic data. We have separately analyzed bathymetric data, which dominates the large scale Earth scaling, and find its large length scaling exponent to be close to 0.5 in agreement with his results. Our analysis, however, indicates a real difference between bathymetric and topographic large scale scaling exponents. The bathymetric data is also rougher on an absolute scale. When considering the entire earth, the inclusion of the continental shelf in Turcotte's analysis makes his absolute roughness still greater. Our results are also consistent with Turcotte's analysis[7] of the scaling behavior of the state of Oregon, for which he reports $\alpha = 0.414$, and the state of Arizona for which he reports $\alpha = 0.41$.

In order to explore the universality of these results, we also study the topography of Venus using Magellan data.[21] Venus data does not suffer from a need to consider oceans. We find that the landscape of Venus is self-affine with $\alpha=0.4$ for length scales between 50 km and 7×10^3 km (Fig. 2). Above 7×10^3 km there are inherent correlations due to the maximum mountain height, determined by the gravitational force and the strength of the substrate. For length scales less than 50 km we find a larger scaling exponent as for the Earth data. We note that care must be taken in analysis of the Venus data because of the existence of deep gorges. Because of these gorges the higher moments of the surface topography have lower scaling exponents; however, the gorges do not contribute substantially to the curves

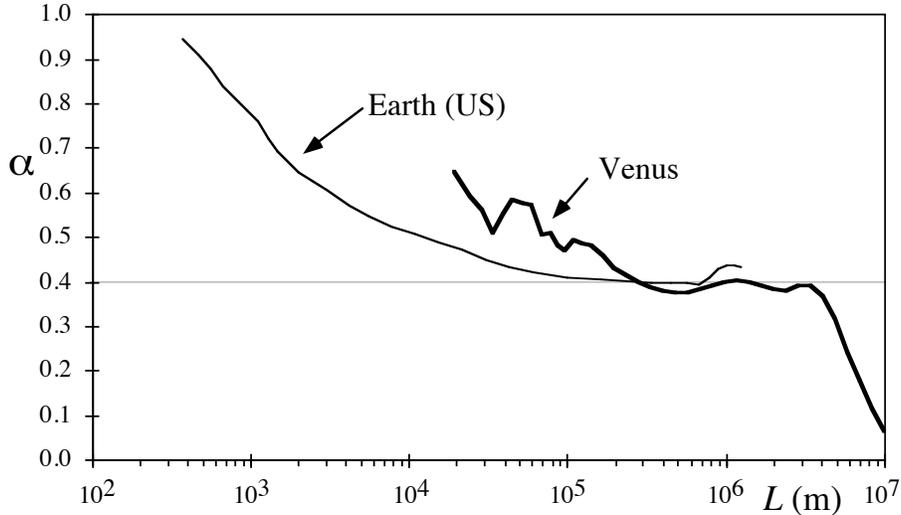


FIG. 3: The roughness scaling as a function of length scale for the continental US and Venus data shown in Fig. 2. The crossover from $\alpha=1$ scaling at small length scales to $\alpha=0.4$ at longer length scales is apparent. The rapid decrease of α for the Venus data at the longest length scales occurs at scales that approach planetary size.

in Fig. 3.

Our results for Venus are also consistent with previous results. Turcotte has analyzed the large scale topography using a spherical harmonic expansion and obtained exponents[16, 22] of $\alpha=0.37$ and $\alpha=0.5$ based on two fits of his data, which has significant scatter. This range is consistent with our results. Turcotte reports that Venus is less rough than the Earth when bathymetric and topographic data are included together. Consistent with the previous discussion we find the Earth topography, limited to the continental US, to be less rough than the Venus topography.

The observation of crossover behavior in both Earth and Venus leads to questions about the mechanisms that cause the crossover and the origin of the characteristic length scales, which are distinct on Earth (5 km) and Venus (50 km). This can only be understood once the processes responsible for the two different regimes are understood. The large scale behavior follows the expected universal scaling for up-down asymmetric dynamic surfaces, and thus might be assumed justified even if detailed mechanisms are not well established. However, the short length behavior must still be understood. It seems natural to attribute the highly correlated smooth behavior of the Earth and Venus at the shortest length scales to

erosional processes. However, recent work[23, 24] has suggested that erosion produces self-affine behavior, which would not be consistent with the observed results. Thus, if erosion is indeed responsible for the short range scaling behavior, the relevant mechanisms of erosion must be better understood.

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