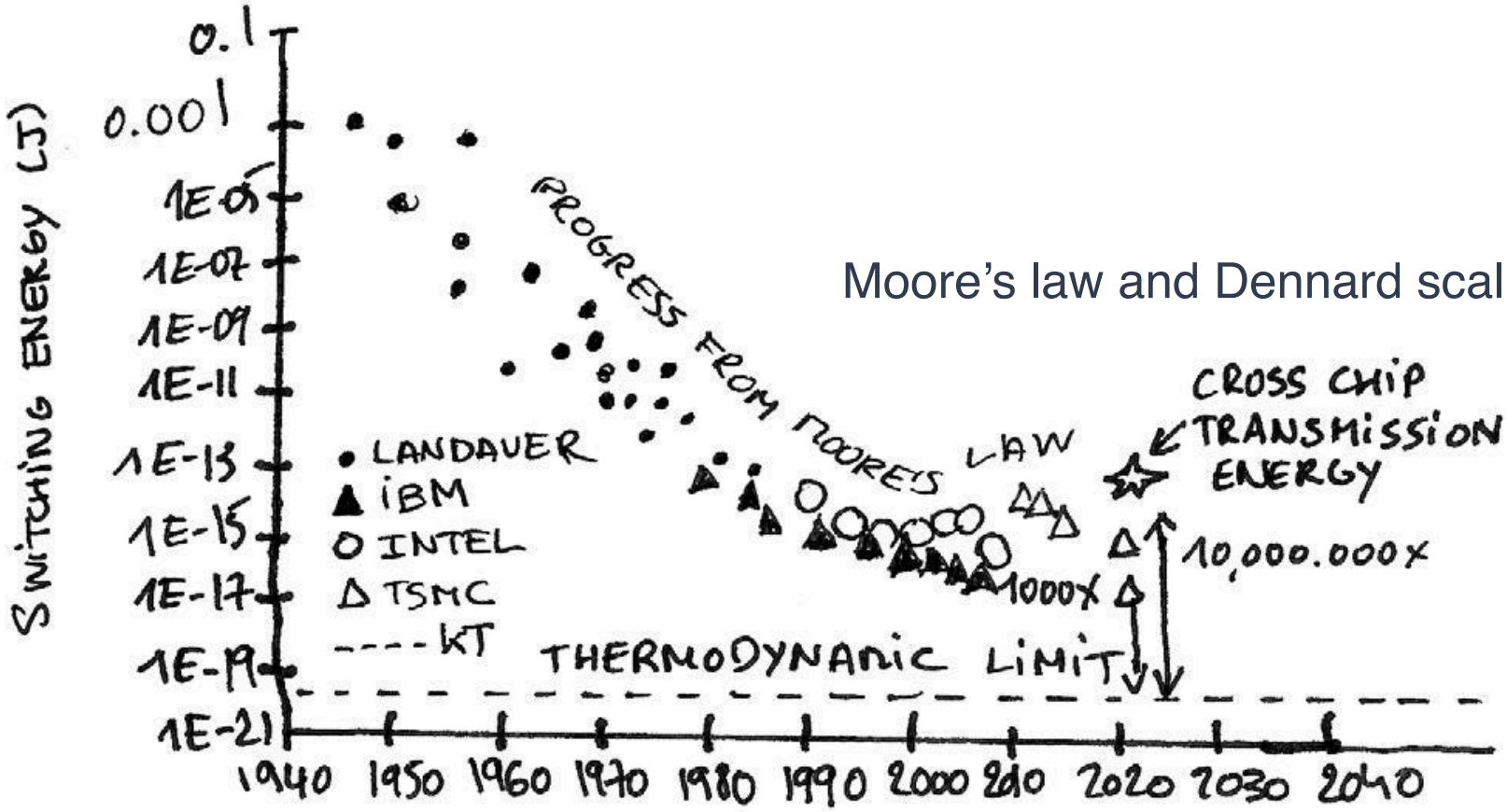
Thermodynamic Linear Algebra

Gavin E. Crooks

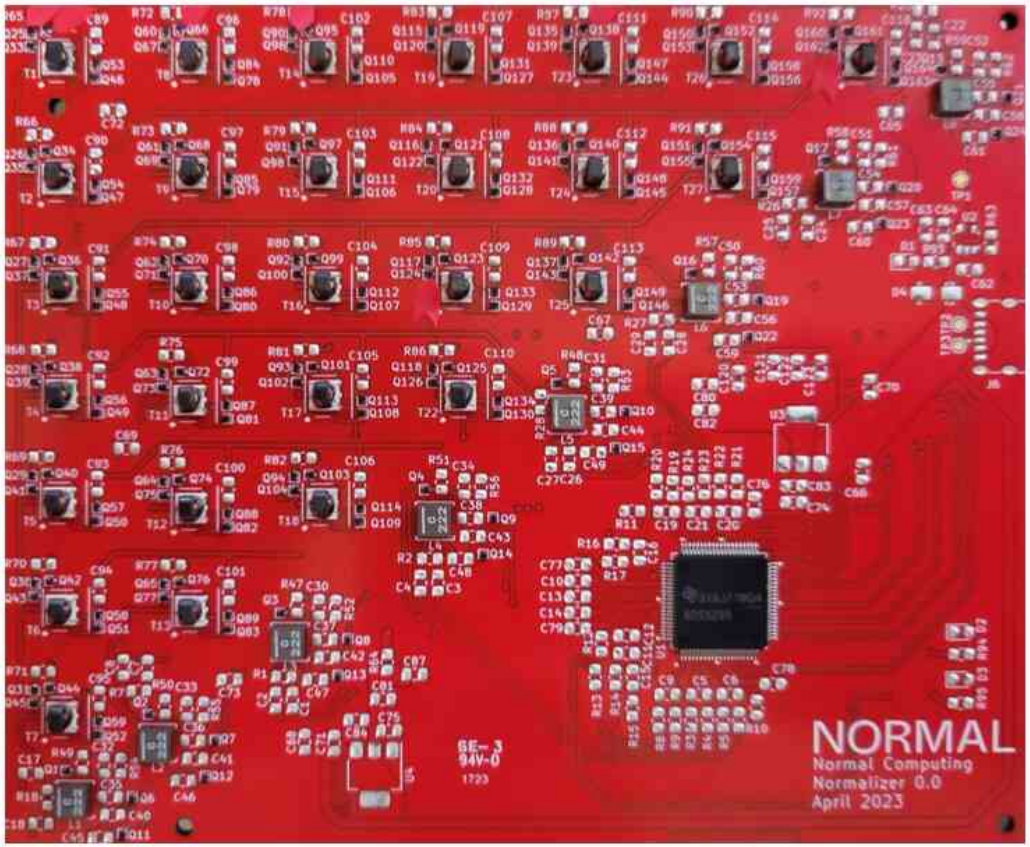
NORMAL Computing

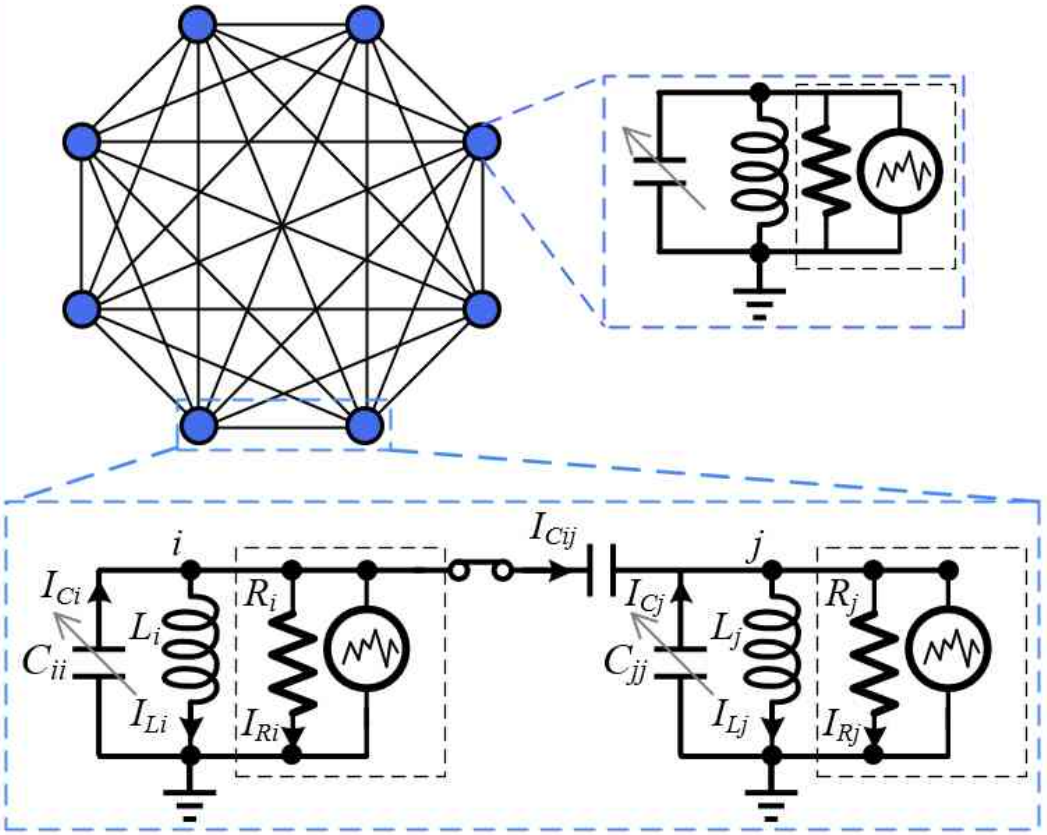
Transistor



Moore's law and Dennard scaling

Stochastic Processing Unit (SPU)





Stochastic Processing Unit Dynamics

Overdamped or Underdamped Langevin dynamics

Currents

$$\mathrm{d}I = \mathbf{L}^{-1} V \mathrm{d}t$$

$$\mathrm{d}V = -\mathbf{C}^{-1}\mathbf{R}^{-1} V \mathrm{d}t - \mathbf{C}^{-1} I \mathrm{d}t + \sqrt{2\kappa_0}\mathbf{C}^{-1}\mathcal{N}[0, \mathbb{I}\,\mathrm{d}t],$$

Voltages

L: Inductances

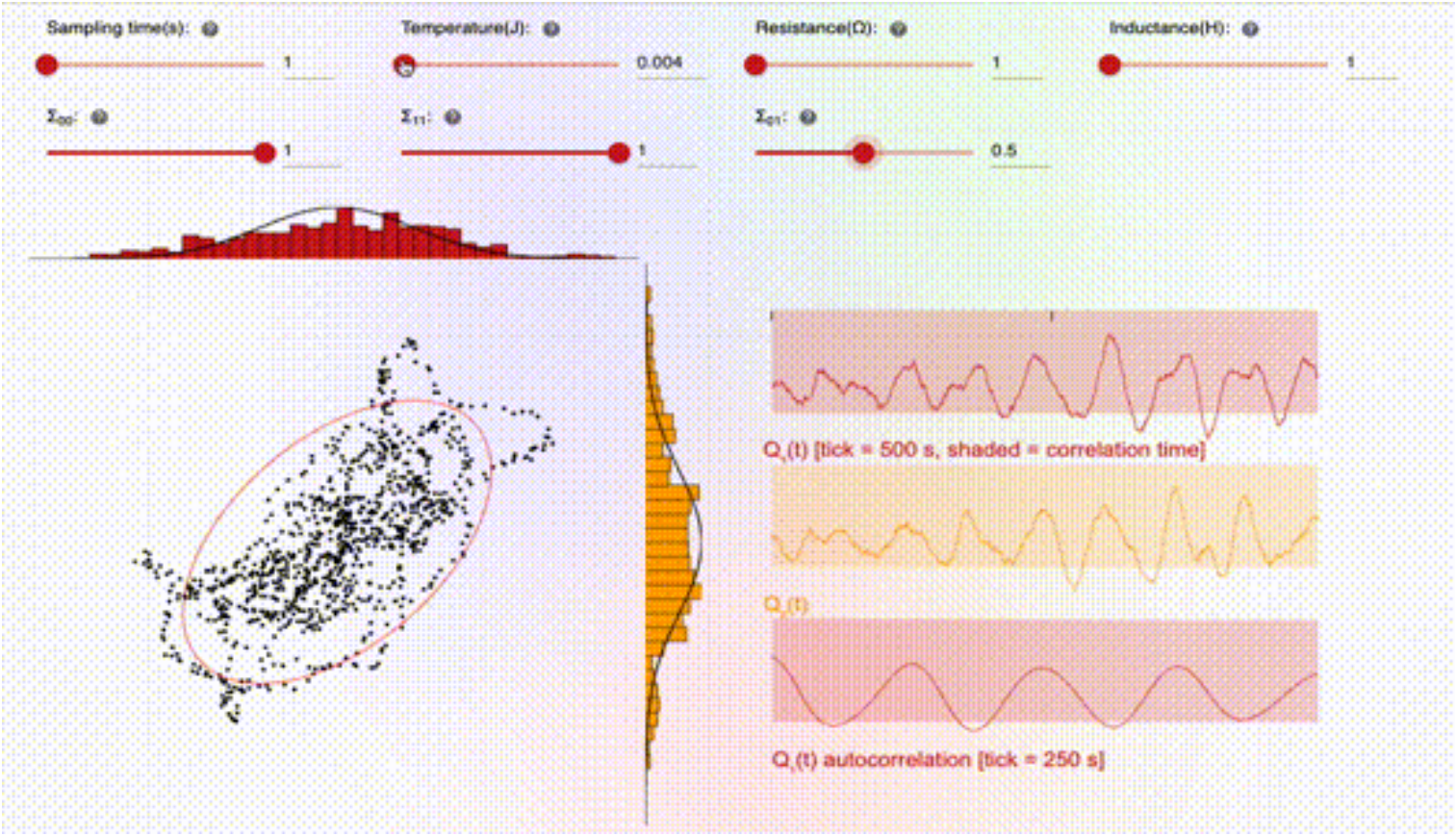
$$\mathcal{H}\left(\vec{I}, \vec{V}\right) = \frac{1}{2}\vec{V}^T \mathbf{C} \vec{V} + \frac{1}{2}\vec{I}^T \mathbf{L} \vec{I},$$

Hamiltonian

C: Maxwell Capacitance Matrix

Thermal Playground

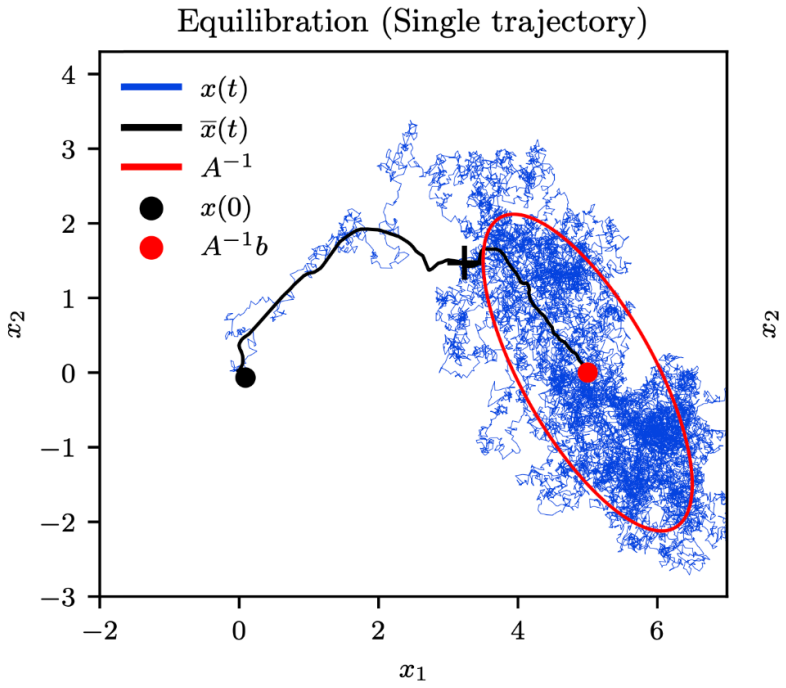
https://app.normalcomputing.ai/composer



Gaussian Sampling with Stochastic Processing Unit

For harmonic oscillator system, at thermal equilibrium, *x* is Gaussian distributed:

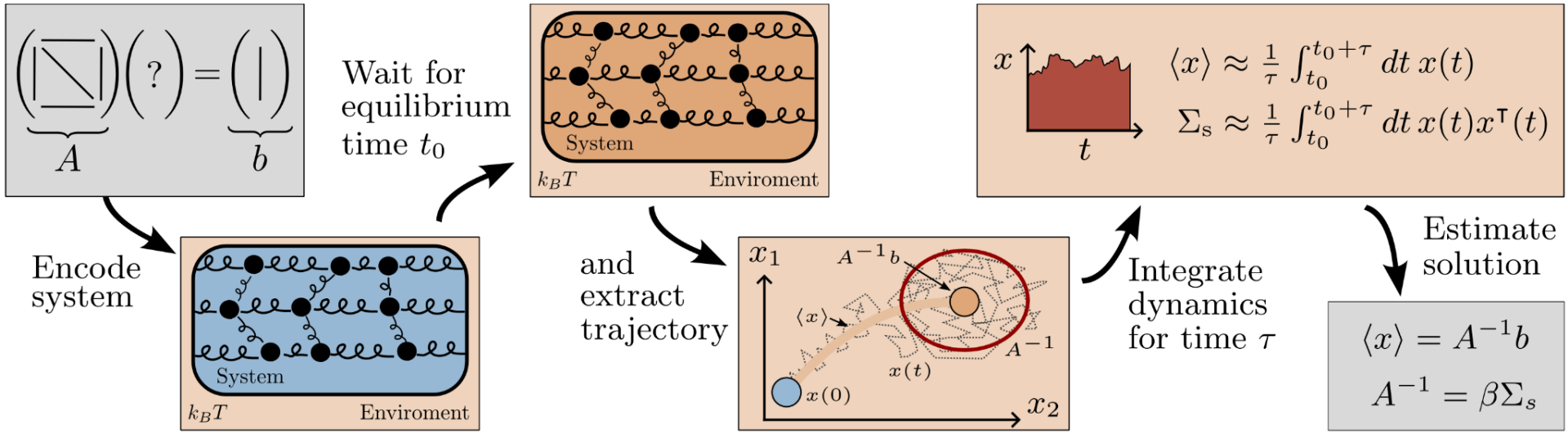
$$\mathcal{N}(\vec{x}|\mathbf{\Sigma}) = \frac{1}{\sqrt{(2\pi)^N |\mathbf{\Sigma}|}} \exp\left(-\frac{1}{2}\vec{x}^T \mathbf{\Sigma}^{-1}\vec{x}\right)$$



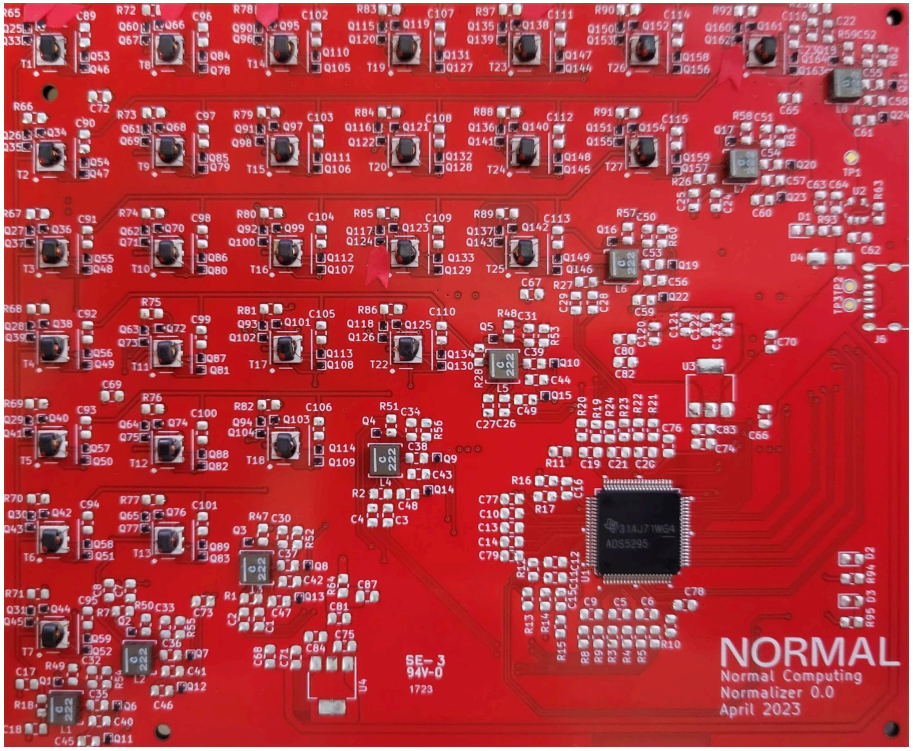
Maxwell capacitance matrix (C) and covariance matrix are related.

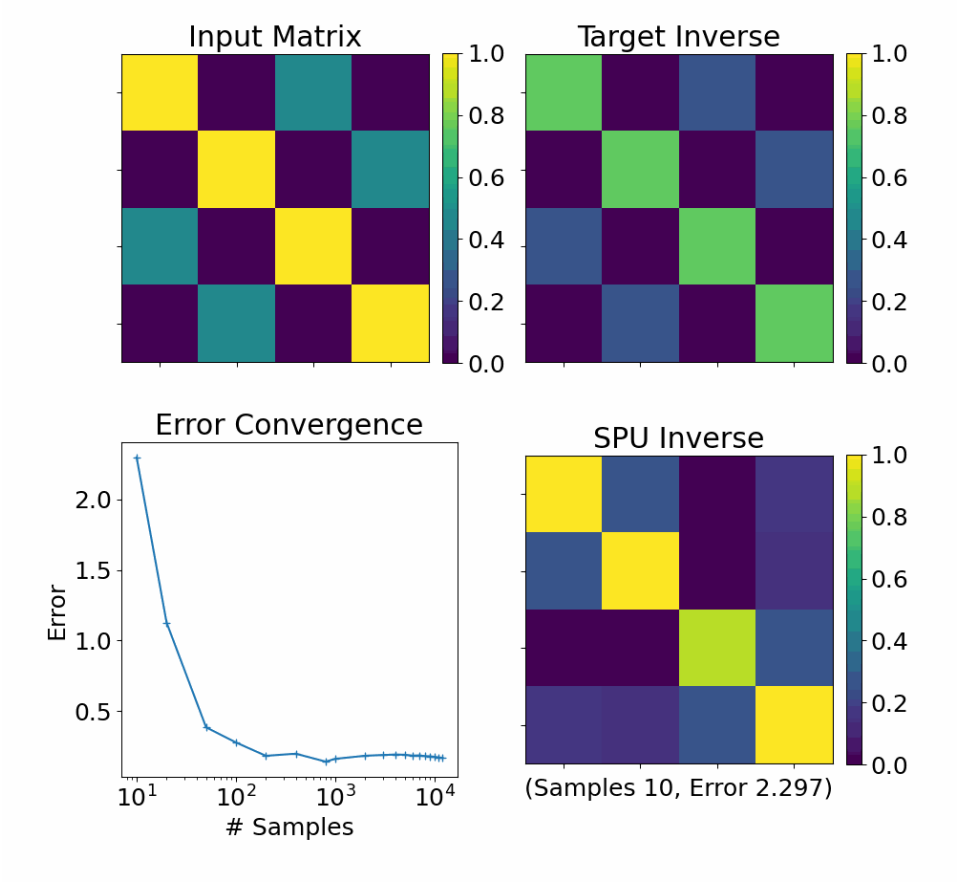
$$C = k_B T\ \Sigma^{-1}$$

Matrix Inversion with Stochastic Processing Unit



Matrix Inversion with Stochastic Processing Unit





https://blog.normalcomputing.ai/posts/2023-11-09-thermodynamic-inversion/thermo-inversion.html

Matrix Determinant with Stochastic Processing Unit

$$f_{\mu;\Sigma}(x) = (2\pi)^{-d/2} \left|\Sigma\right|^{-1/2} \exp\left(-\frac{1}{2} x^{\intercal} \Sigma^{-1} x\right),$$

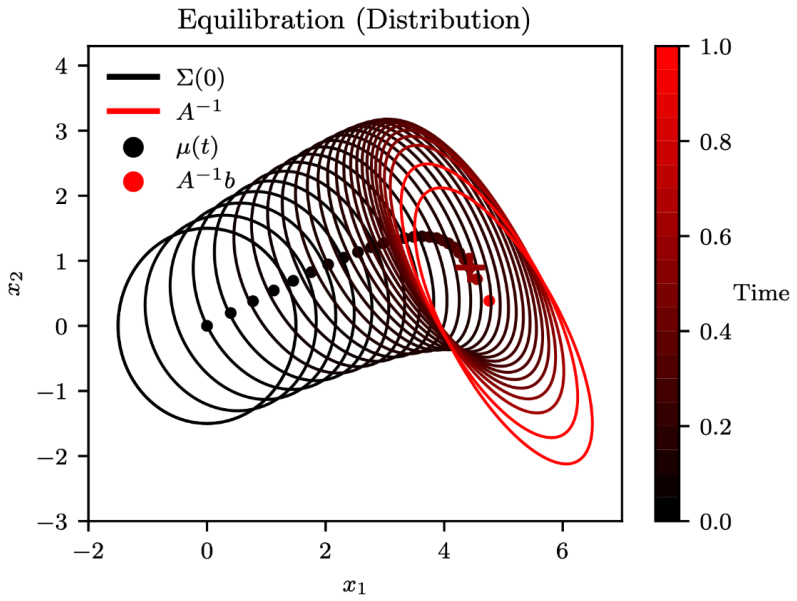
$$S(\Sigma) = \frac{1}{2} \ln\left|\Sigma\right| + \frac{d}{2}(1 + \ln 2\pi)$$

$$\Delta F = \Delta E - \beta^{-1} \Delta S$$

$$\Delta F = -\beta^{-1} \ln\left(\sqrt{\frac{\left|A_2^{-1}\right|}{\left|A_1^{-1}\right|}}\right) = -\beta^{-1} \ln\left(\sqrt{\frac{\left|A_1\right|}{\left|A_2\right|}}\right).$$

Matrix Determinant with Stochastic Processing Unit

$$\Delta F = -\beta^{-1} \ln \left(\sqrt{\frac{|A_2^{-1}|}{|A_1^{-1}|}} \right) = -\beta^{-1} \ln \left(\sqrt{\frac{|A_1|}{|A_2|}} \right)$$



$$e^{-\beta \Delta F} = \langle e^{-\beta W} \rangle$$

Optimal Control and
Thermodynamic Geometry

NORMAL Computing

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Thermodynamic Computing System for AI Applications
arXiv:2312.04836

Thermodynamic Matrix Exponentials
arXiv:2311.12759

Thermodynamic Linear Algebra
arXiv:2308.05660

Thermodynamic AI and the fluctuation frontier
arXiv:2302.06584